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Effect of thermal radiation on unsteady mixed convection flow and heat transfer over a porous stretching surface in porous medium

S. Mukhopadhyay *

Department of Mathematics, M.U.C. Women's College, Burdwan, WB 713 104, India

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ABSTRACT

An analysis is performed to investigate the effects of thermal radiation on unsteady boundary layer mixed convection heat transfer problem from a vertical porous stretching surface embedded in porous medium. The fluid is assumed to be viscous and incompressible. Numerical computations are carried out for different values of the parameters involved in this study and the analysis of the results obtained shows that the flow field is influenced appreciably by the unsteadiness parameter, mixed convection parameter, parameter of the porous medium and thermal radiation and suction at wall surface. With increasing values of the unsteadiness parameter, fluid velocity and temperature are found to decrease in both cases of porous and non-porous media. Fluid velocity decreases due to increasing values of the parameter of the porous medium resulting an increase in the temperature field in steady as well as unsteady case.

1. Introduction

The heat, mass and momentum transfer in the laminar boundary layer flow on a stretching sheet are important from theoretical as well as practical point of view because of their wider applications to polymer technology and metallurgy. The thermal buoyancy force arising due to the heating of stretching surface, under some circumstances, may alter significantly the flow and thermal fields and thereby the heat transfer behaviour in the manufacturing processes. Keeping this fact in mind, Lin et al. [1], Chen [2], Ali and Al-Youself [3] etc. investigated the flow problems considering the buoyancy force.

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and under ground energy transport. A very significant area of research in radiative heat transfer, at the present time is the numerical simulation of combined radiation and convection/conduction transport processes (Kandasamy et al. [4]). We know that the radiation effect is important under many non-isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic.

All of the above mentioned studies consider the steady-state problem. But, in certain practical problems, the motion of the stretched surface may start impulsively from rest. In these problems, the transient or unsteady aspects become more interesting. Recently, Elbashbeshy and Bazid [5] presented an exact similarity solution for unsteady momentum and heat transfer flow whose motion is caused solely by the linear stretching of a horizontal stretching surface. Since no attempt has been made to analyse the effects of thermal radiation on heat and mass transfer on unsteady boundary layer mixed convection flow over a vertical stretching surface in porous medium in presence of suction, this problem is investigated in this article. The momentum and the thermal boundary layer equations are solved using shooting method and the numerical calculations were carried out for different values of parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of unsteadiness, heat radiation, mixed convection and suction on the wall in presence of porous medium. To reveal the tendency of the solutions, representative results are presented for the velocity, temperature as well as the skin friction and rate of heat transfer. Comparisons with previously published works are performed and excellent agreement between the results is obtained.

2. Equations of motion

We consider the two-dimensional mixed convection boundary-layer flow of an incompressible viscous liquid through porous

* Tel.: +91 342 2557741; fax: +91 342 2530452.
E-mail address: swati_bumath@yahoo.co.in

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Nomenclature

c	constant
c_p	specific heat at constant pressure
$Da_x = \frac{k_1(1-\alpha t)}{x^2}$	local Darcy number
f	non-dimensional stream function
f', f'', f'''	first order, second order, third order derivatives respectively with respect to η
$Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^3}$	Grashof number
g	gravity field
k	permeability of the porous medium
k^*	absorption coefficient
$M = \frac{\alpha}{c}$	unsteadiness parameter
$N = \frac{\kappa k^*}{4\sigma T_\infty^3}$	radiation parameter
Pr	Prandtl number
p, q	variables
q_r	radiative heat flux
$Re_x = \frac{u_w x}{\nu}$	local Reynold's number
$S(>0)$	suction parameter
T	temperature of the fluid
T_w	temperature of the wall of the surface
T_∞	free-stream temperature
u, v	components of velocity in the x and y directions
$v_0(>0)$	velocity of suction of the fluid
z	variable

Greek symbols

α	constant
β	volumetric coefficient of thermal expansion
η	similarity variable
κ	coefficient of thermal diffusivity
$\lambda = \frac{Gr_x}{Re_x^2}$	mixed convection parameter
μ	dynamic viscosity
ν	kinematic viscosity
ψ	stream function
ρ	density of the fluid
σ	Stefan-Boltzman constant
θ	non-dimensional temperature
θ', θ''	first order, second order derivatives respectively with respect to η

medium along a permeable vertical wall stretching with velocity $u_w = \frac{cx}{1-\alpha t}$ and with temperature distribution $T_w = T_\infty + 1/2 T_0 Re_x x^{-1}(1-\alpha t)^{-1} = T_\infty + T_0 \frac{cx}{2\nu}(1-\alpha t)^{-2}$ (Andersson et al. [6]) where $Re_x = \frac{u_w x}{\nu}$ is the local Reynold's number. The x-axis is directed along the stretching surface and points in the direction of motion. The y-axis is perpendicular to it. u, v are the velocity components in the x- and y-directions. The governing equations under boundary layer and Boussinesq approximations for flow through a porous medium over the stretching surface are, in the usual notation may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) - \frac{\nu}{k}u, \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{3}$$

along with the boundary conditions

$$u = u_w = \frac{cx}{1-\alpha t},\ v = v_w = -\frac{v_0}{(1-\alpha t)^{\frac{1}{2}}},\ T = T_w \quad \text{at} \quad y = 0, \tag{4}$$

$$u \to 0, T \to T_\infty \quad \text{as} \quad y \to \infty. \tag{5}$$

Here $k[=k_1(1\text{-}\alpha t)]$ is the permeability of the porous medium, k_1 is the initial permeability, μ is the coefficient of fluid viscosity, ρ is the fluid density, $\nu = \mu/\rho$ is the kinematic viscosity, β is the volumetric coefficient of thermal expansion, g is the gravity field, T is the temperature, κ is the coefficient of thermal conductivity of the fluid, $v_0(>0)$ is the velocity of suction of the fluid, $c(>0)$ and $\alpha(>0)$ are constants with dimension $(\text{time})^{-1}$, T_w is the uniform wall temperature, T_∞ is the free-stream temperature, c_p is the specific heat at constant pressure and q_r is the radiative heat flux. The viscous dissipative term in the energy equation is neglected here.

Using Rosseland approximation, we get $q_r = -\frac{4\sigma}{3k^*}\frac{\partial T^4}{\partial y}$ where σ is the Stefan-Boltzman constant, k^* is the absorption coefficient. We assume that the temperature difference within the flow is such that T^4 may be expanded in a Taylor's series. Expanding T^4 about T_∞ and neglecting higher orders we get, $T^4 = 4T_\infty^3 T - 3T_\infty^4$.

Now Eq. (3) becomes

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho c_p} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial y^2}. \tag{6}$$

2.1. Method of solution

We now introduce the following relations for u, v and θ as

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x} \quad \text{and} \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}$$

where ψ is the stream function.

Using the relation (7) in the boundary layer Eq. (2) and in the energy Eq. (6) we get the following equations

$$\frac{\partial^2 \psi}{\partial t \partial y} + \frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2} = \nu\frac{\partial^3 \psi}{\partial y^3} + g\beta(T_w - T_\infty)\theta - \frac{\nu}{k}\frac{\partial \psi}{\partial y} \tag{8}$$

and

$$\frac{\partial T}{\partial t} + \frac{\partial \psi}{\partial y}\frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial T}{\partial y} = \left(\frac{\kappa}{\rho c_p} + \frac{16\sigma T_\infty^3}{3\rho c_p k^*}\right)\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

We now introduce the similarity variable η and the dimensionless variables f and θ as follows:

$$\eta = \left(\frac{c}{\nu(1-\alpha t)}\right)^{\frac{1}{2}} y, \psi = \left(\frac{\nu c}{(1-\alpha t)}\right)^{\frac{1}{2}} x f(\eta),\ T = T_\infty + T_0\frac{cx}{2\nu}(1-\alpha t)^{-2}\theta(\eta). \tag{10}$$

In view of the relations (10), the Eqs. (8) and (9) become

$$M\left(\frac{\eta}{2}f'' + f'\right) + f'^2 - ff'' = f''' + \lambda\theta - \frac{1}{D}f', \tag{11}$$

$$\frac{M}{2}(\eta\theta' + 4\theta) + f'\theta - f\theta' = \frac{1}{Pr}\left(1 + \frac{4}{3N}\right)\theta'', \tag{12}$$

where $\lambda = \frac{g\beta T_0}{2\nu c} = \frac{Gr_x}{Re_x^2}$ is the mixed convection parameter, $Gr_x = \frac{g\beta(T_w - T_\infty)x^3}{\nu^3}$ is the Grashof number, $D = Da_x Re_x = \frac{k_1 c}{\nu}$, $Da_x = \frac{k}{x^2} = \frac{k_1(1-\alpha t)}{x^2}$ is the local Darcy number, $M = \frac{\alpha}{c}$ is the unsteadiness parameter, $N = \frac{\kappa k^*}{4\sigma T_\infty^3}$ is the radiation parameter.

The boundary conditions are transformed to

$$f'(\eta) = 1, f(\eta) = S \quad \text{and} \quad \theta(\eta) = 1 \quad \text{at} \quad \eta = 0, \tag{13}$$
$$f'(\eta) \to 0, \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \tag{14}$$

where $S = \frac{v_0}{(\nu c)^{\frac{1}{2}}} (> 0)$ is the suction parameter.

3. Numerical method for solution

The above Eqs. (11) and (12) along with boundary conditions are solved by converting them to an initial value problem. We set

$$f' = z, z' = p, p' = M\left(\frac{\eta}{2}p + z\right) + z^2 - fp - \lambda\theta + D^{-1}z. \tag{15}$$

$$\theta' = q, q' = \frac{3NPr}{4+3N}\left(\frac{M}{2}(\eta q + 4\theta) + z\theta - fq\right) \tag{16}$$

with the boundary conditions

$$f(0) = S, f'(0) = 1, \theta(0) = 1. \tag{17}$$

In order to integrate (15) and (16) as an initial value problem we require a value for $p(0)$ i.e. $f''(0)$ and $q(0)$ i.e. $\theta'(0)$ but no such values are given in the boundary. The suitable guess values for $f''(0)$ and $\theta'(0)$ are chosen and then integration is carried out. We compare the calculated values for f' and θ at $\eta = 12$ (say) with the given boundary conditions $f'(12) = 0$ and $\theta(12) = 0$ and adjust the estimated values, $f''(0)$ and $\theta'(0)$, to give a better approximation for the solution.

We take a series of values for $f''(0)$ and $\theta'(0)$ and apply the fourth order classical Runge-Kutta method with step-size $h = 0.01$. The above procedure is repeated untill we get the results up to the desired degree of accuracy, 10^{-5}.

4. Results and discussions

Computation through employed numerical scheme has been carried out for various values of the parameters such as unsteadiness parameter M, radiation parameter N, suction parameter S, mixed convection parameter λ, permeability parameter D^{-1} and Prandtl number Pr. For illustrations of the results, numerical values are plotted in the figures.

In order to validate the method, comparison with available steady state results of Grubka and Bobba [7] and Chen [2] for local Nusselt number $Nu_x Re_x^{\frac{-1}{2}} = -\theta'(0)$ for forced convection flow on a linearly stretching surface in the absence of porous medium, wall suction and thermal radiation are made (Table 1) and found in excellent agreement.

First, we present the result for variation of the parameter M taking $N = 0.1, S = 0.1, \lambda = 0.1, D^{-1} = 0.1, Pr = 0.5$. In Fig. 1, velocity profiles are shown for different values of M. It is seen that the horizontal velocity decreases with the increase of unsteadiness parameter M. It is evident from this figure that the thickness of the boundary layer decreases with the increasing values of M.

Fig. 2 represents the temperature profiles for different values of $M(= 0.1, 0.3, 0.5)$. For all values of M considered, θ is found to decrease with the increase of η. Significant change in the rate of decrease of θ for increasing values of M is noticed. Temperature at a point on the sheet decreases significantly with the increase in M i.e. rate of heat transfer increases with increasing unsteadiness parameter M. Same results are obtained in case of non-porous media also.

Next, we present the effects of thermal radiation on velocity and temperature profiles. Figs. 3 and 4 are the graphical representations of horizontal velocity profile $f'(\eta)$ and temperature profile $\theta(\eta)$ for the different values of the radiation parameter N in case of porous media. It is found that horizontal velocity $f'(\eta)$ decreases as the radiation parameter N increases (Fig. 3). Temperature $\theta(\eta)$ decreases as thermal radiation increases (Fig. 4). This is in agreement with the physical fact that the thermal boundary layer thickness decreases with increasing N.

Figs. 5 and 6 display the effects of suction parameter (S) on velocity and temperature profiles. Fluid velocity and the tempera-

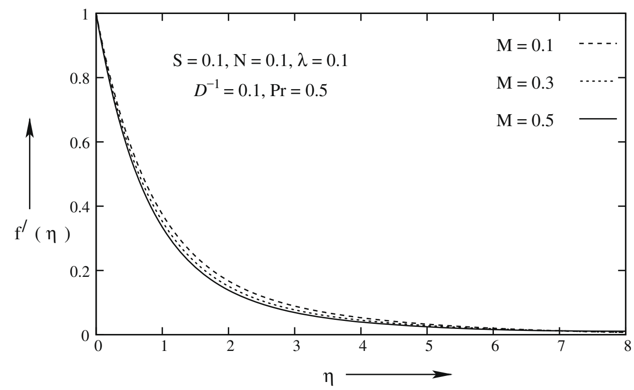

Fig. 1. Velocity profiles for variable unsteadiness parameter *M*.

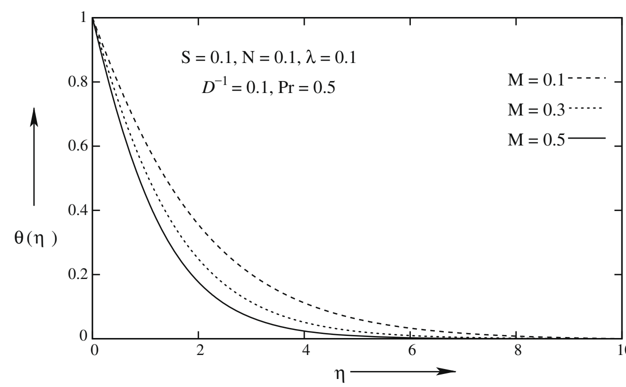

Fig. 2. Temperature profiles for variable unsteadiness parameter *M*.

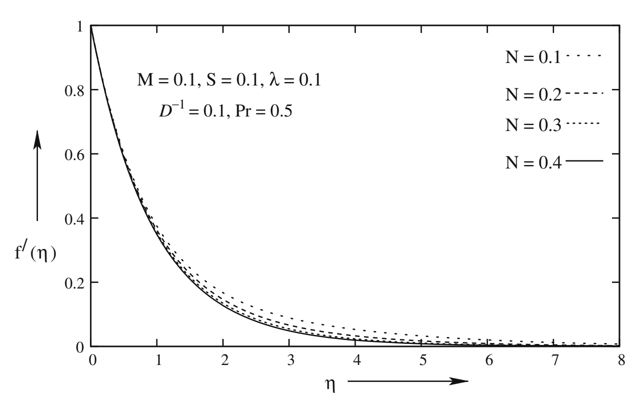

Fig. 3. Velocity profiles for variable radiation parameter *N*.

Table 1
Values of $Nu_x Re_x^{\frac{-1}{2}}$ for several values of Pr with $M = 0, \lambda = 0, N = 0, S = 0$.

Pr	Grubka and Bobba [7]	Chen [2]	Present study
0.01	0.0294	0.02942	0.02944
0.72	1.0885	1.08853	1.08855
1.00	1.3333	1.33334	1.33334
3.00	2.5097	2.50972	2.50971

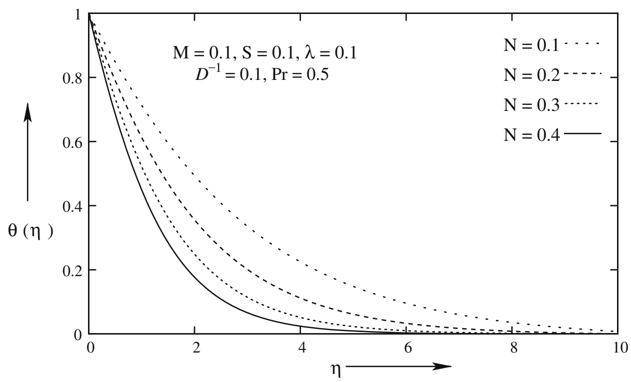

Fig. 4. Temperature profiles for variable radiation parameter N.

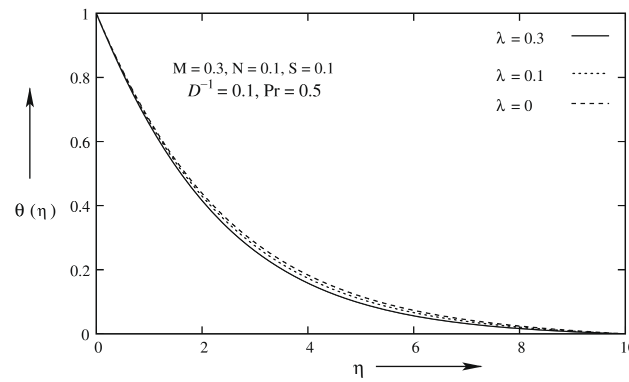

Fig. 8. Temperature profiles for variable values of mixed convection parameter λ.

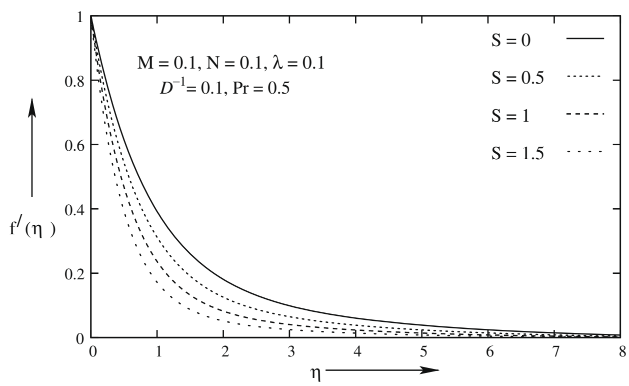

Fig. 5. Velocity profiles for variable values of suction parameter S.

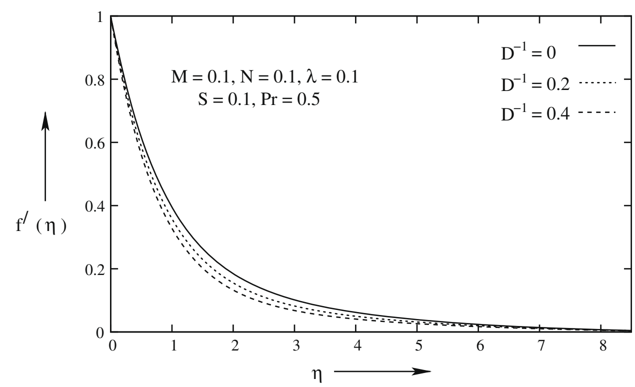

Fig. 9. Velocity profiles for variable values of permeability parameter (D^{-1}).

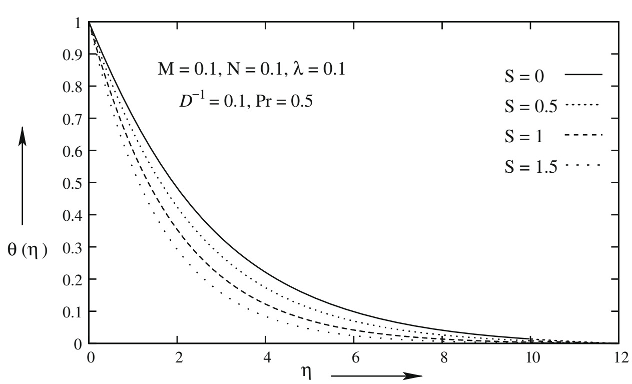

Fig. 6. Temperature profiles for variable values of suction parameter S.

ture of the sheet both are found to decrease with increasing values of S. The physical explanation for such behaviour is as follows. In this case, the heated fluid is pushed towards the wall where the buoyancy forces can act to retard the fluid due to high influence of viscosity. This effect acts to decrease the wall shear stress. Thermal boundary layer thickness reduces in case of suction.

Now we focus our attention on the behaviour of fluid velocity and the temperature of the sheet due to increase of the mixed convection parameter λ. Fluid velocity increases with increasing values of λ (Fig. 7) but the temperature decreases in this case (Fig. 8). Same behaviour is noted in case of non-porous media also.

Fluid flow and heat transfer towards a porous stretching sheet have an important bearing on several technological processes. Figs. 9 and 10 represent the influences of permeability parameter on flow velocity and temperature. It is obvious that the presence of

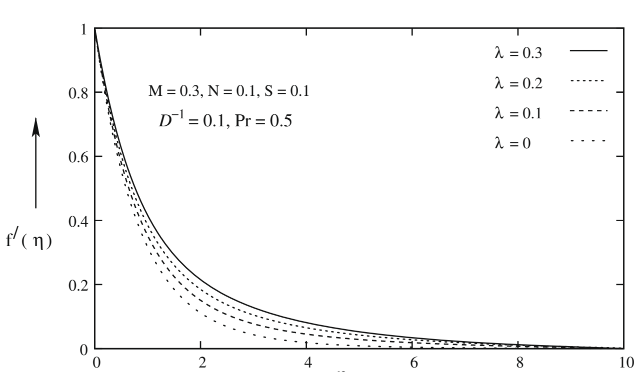

Fig. 7. Velocity profiles for variable values of mixed convection parameter λ.

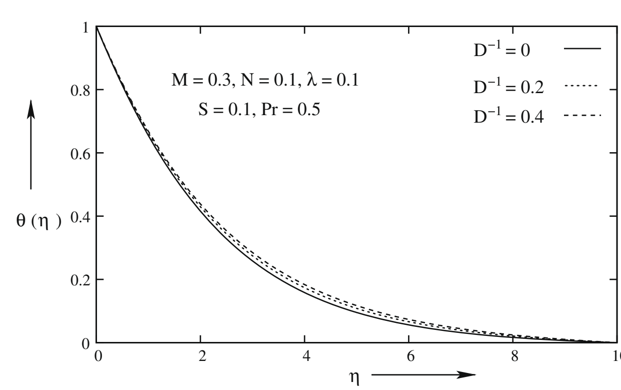

Fig. 10. Temperature profiles for variable values of permeability parameter (D^{-1}).

porous medium causes higher restriction to the fluid, which reduces the fluid velocity (Fig. 9) and enhanced the temperature (Fig. 10). Here $D^{-1} = 0$ indicates the non-porous media.

5. Conclusion

The purpose of this paper is to present numerical solutions of unsteady boundary layer flow and heat transfer on a permeable stretching sheet embedded in a porous medium taking into consideration, the effect of buoyancy and thermal radiation. With the help of similarity transformations, the governing time dependent boundary layer equations for momentum and thermal are reduced to coupled ordinary differential equations which are then solved numerically using shooting method. The results pertaining to the present study indicate that the flow and temperature field are significantly influenced by the unsteadiness parameter, buoyancy force, suction parameter in both porous and non-porous media. The effect of increasing values of permeability parameter on viscous incompressible liquid is to suppress the velocity field. This, in turn, causes the enhancement of the temperature field.

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